Partitioning Bipartite Graphs: A Modified Louvain

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How do we find communities in a graph? How does this change if the graph is bipartite? The Louvain method maximizes links within communities and minimizes those between in order to determine an optimal grouping. Yet, because it may fail when bipartite restrictions are introduced, we have adjusted the null model so as to improve performance in these conditions.

The Louvain Algorithm

- Maximizes modularity coefficient, -1 to 1

$$Q=\frac{1}{2m}\sum_{i,j}\left[A_{ij}-\frac{k_i k_j}{2m}\right]\delta(c_i,c_j)$$

m= number of edges

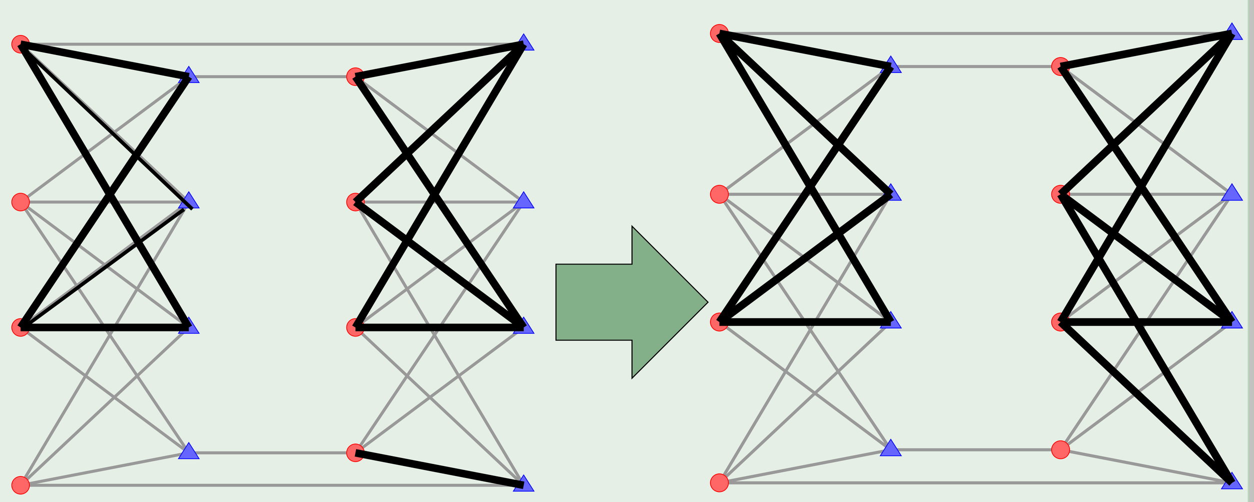
A_{ij} = 1 iff nodes *i* and *j* are connected

k_i, k_j are the degrees of nodes *i* and *j*

$\frac{k_i k_j}{2m}$ indicates probability of edge

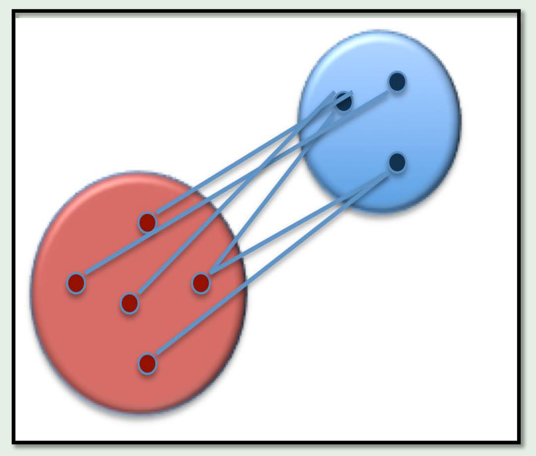
$\delta(c_i,c_j)=1$ iff nodes *i* and *j* are in the same community

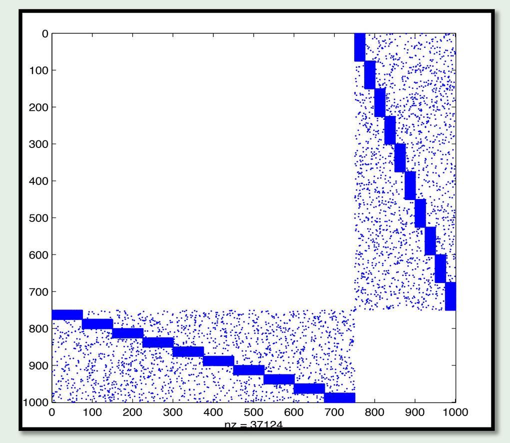
Updating Procedure



- Bold edges indicate that the connected nodes are in the same community
- Each node starts in its own community
- What move (or lack of) most improves modularity?
- In next pass, treats each community as a node
- Sweeps through until no moves remain that increase modularity

What if the Graph is Bipartite?





Adjusted Null Model

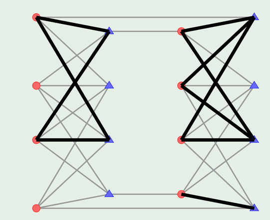
Original	Bipartite
$A-\begin{bmatrix}\frac{rr^t}{2m} & \frac{rb^t}{2m}\\ \frac{br^t}{2m} & \frac{bb^t}{2m}\end{bmatrix}$	$A-\begin{bmatrix}0 & \frac{rb^t}{m}\\ \frac{br^t}{m} & 0\end{bmatrix}$

- Doesn't consider impossible connections
- Half the number of edges
- Bipartite modularity coefficient now defined as:

$$Q=\frac{1}{m}\sum_{i,j}\left[A_{ij}-\frac{k_i k_j}{m}\right]\delta(c_i,c_j)\xi(g_i,g_j)$$

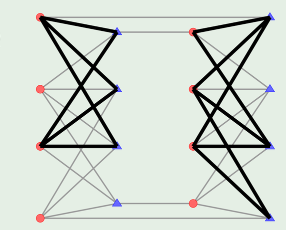
$\xi(g_i,g_j)=1$ iff nodes *i* and *j* are of different colors

A Small Example



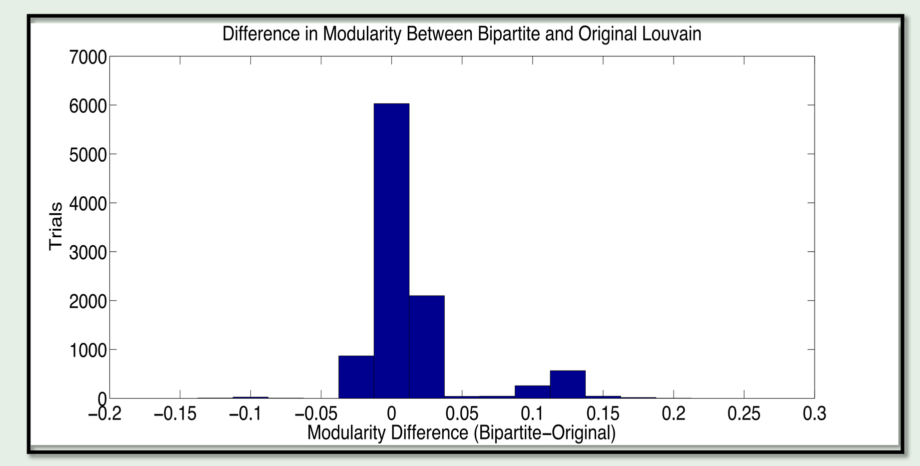
16 nodes: 8 red, 8 blue

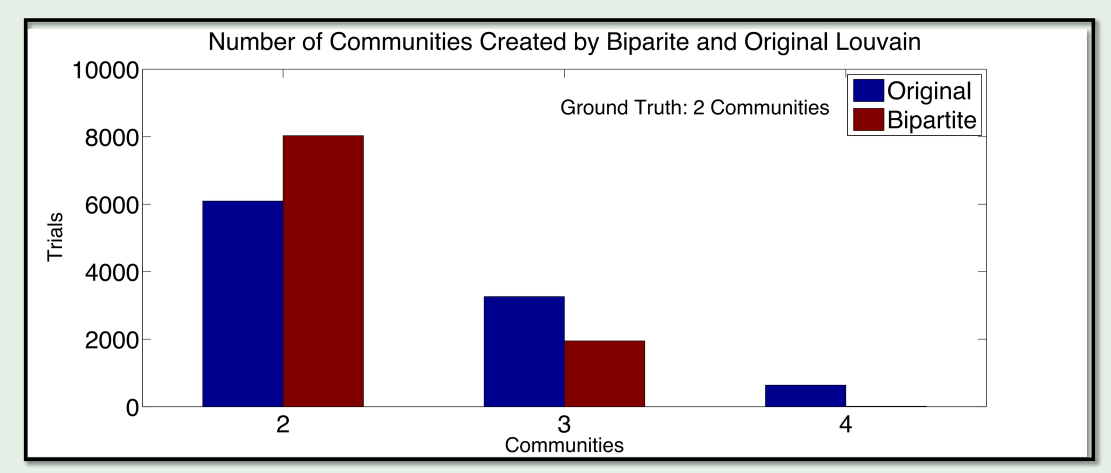
4 degrees each



Long diagonals removed and replaced with edges between cliques

10000 permutations of the nodes





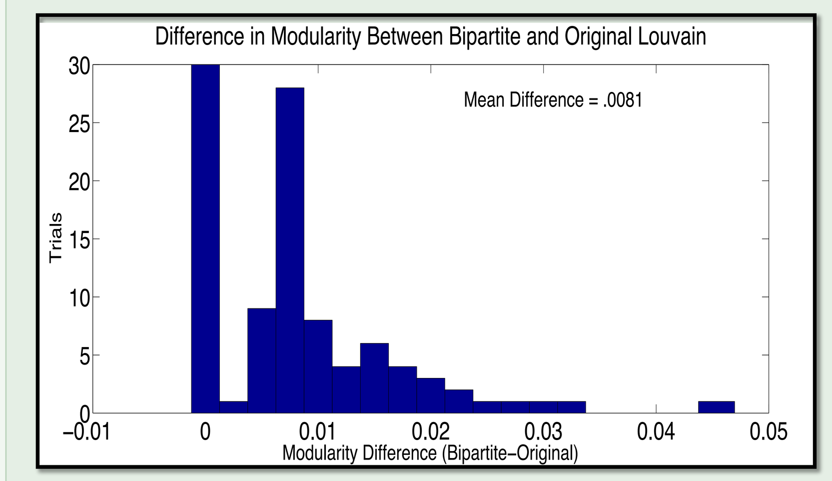
Results: Bipartite Louvain improved modularity by an average of .0145 and correctly detected two communities 80% of the time as compared to 61% by standard Louvain.

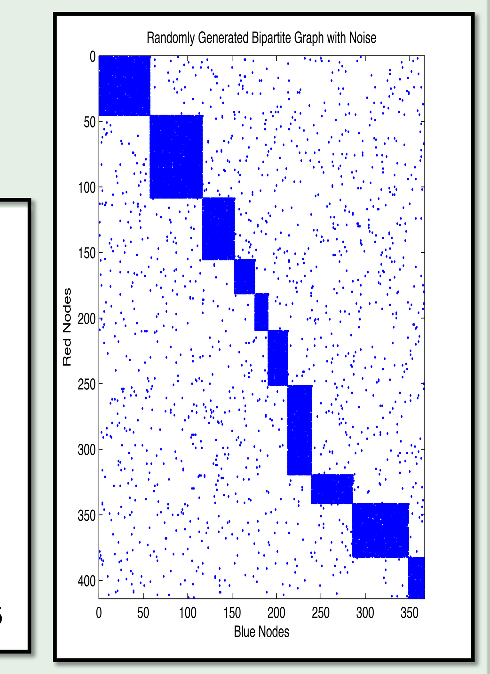
A Large Example

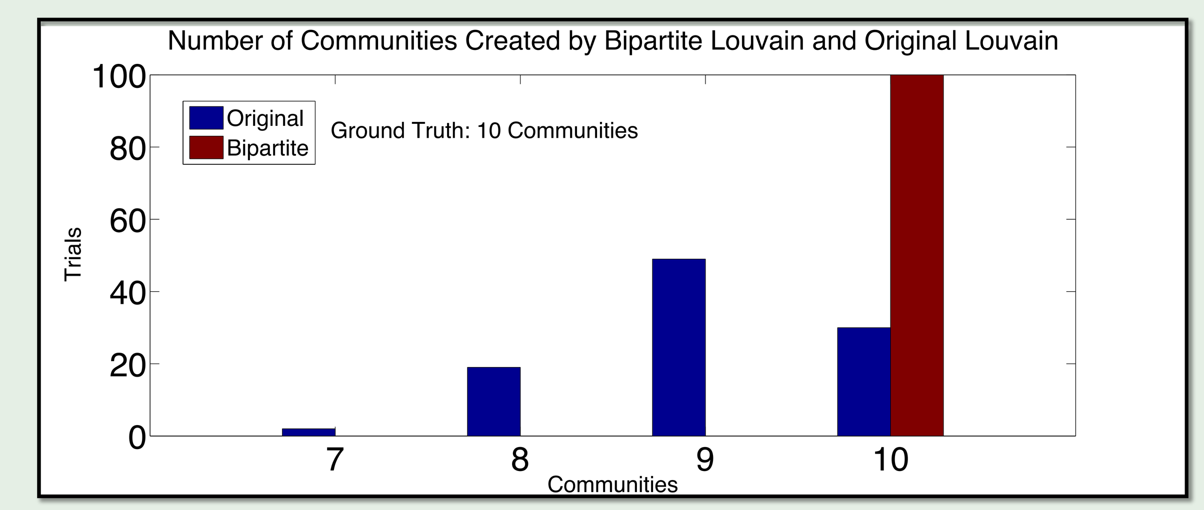
Randomly generated graph with noise

779 vertices: 413 red, 366 blue

100 permutations of the nodes







Results: Bipartite Louvain improved modularity by an average of .0081 and correctly detected 10 communities in every case as compared to 30% by standard Louvain.

Conclusion and Future Work

Our Bipartite Louvain is more robust with respect to permutations of vertices than the standard Louvain. For our synthetic examples, Bipartite Louvain typically yields a higher modularity and uncovers the ground truth communities with a higher probability. In the future, we will examine real world data sets with our modified algorithm.

References

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This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

LLNL-POST-658297